Asymmetric Nuclear Matter in the Relativistic Brueckner Hartree Fock approach

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Abstract

We present a calculation of asymmetric nuclear matter properties in a relativistic Brueckner Hartree Fock framework. Following other calculations the components of the self-energies are extracted by projecting on Lorentz invariant amplitudes. It is shown that for asymmetric nuclear matter one needs a sixth invariant. We present a set of invariants which in the limit of symmetric nuclear matter reduces to the conventional set. We argue that the existence of such a properly behaving set is also crucial for the application of the projection method in symmetric nuclear matter. Results for the equation of state and other observables are presented. Special attention is payed to an analysis in terms of mean-field effective coupling constants. Apart from the usual ones we also find significant strength in the isovector scalar channel, which can be interpreted as an effective δ-meson.

21.65.+f, 21.60.-n

I. INTRODUCTION

An important step in understanding the binding properties of nuclear matter was made with relativistic approaches to nuclear interactions. They were capable to resolve to a large extent the longstanding problem of the underestimation of the binding energy of nuclear matter found in non-relativistic calculations. Conceptually, the relativistic theory provides a very natural link to modern boson exchange interactions and - at least in principle - relates in-medium interactions to free space nucleon-nucleon (NN) scattering.

Applications have mostly been concentrating on the description of symmetric nuclear matter [1–5]. Although being different in theoretical and numerical details the various approaches are in rather close agreement on the bulk properties of nuclear matter. Investigations of asymmetric nuclear matter, however, are rare, most of them of a very recent date [6–9]. One reason is certainly that up to very recent years the data base on asymmetric systems was rather limited because stable nuclei range around $Z/A \simeq 0.5$ with a rather small bandwidth of variations. Asymmetric matter was mainly of theoretical interest, except for astrophysical studies, as the structure of neutron stars [10]. The situation has changed completely with the new radioactive beam facilities producing now new isotopes in a sufficiently large amount for detailed studies. The isospin degree of freedom in the nuclear equation of state can be investigated for the first time over a much larger range of proton-neutron asymmetries as before. At present, the largest asymmetries produced experimentally are obtained for light dripline nuclei ($Z/A = 0.2 \cdots 0.3$ for the heavy isotopes of He to oxygen nuclides) which is about a factor of 2 less of the asymmetries found in stable nuclei. In the near future an increasing amount of data for nuclides all over the mass table can be expected.

It is obvious that the properties of asymmetric matter are of high interest for nuclear structure at the proton and neutron driplines and *vice versa*. The isospin dependence of the nuclear equation of state is to a large extent unexplored. Extrapolations of the nuclear mass formula into the regions of new isotopes persistently result in strong deviations from measured binding energies. The same is found for structure calculations with seemingly well established effective interaction models as the non-relativistic Skyrme energy functional. Although the parameters of Skyrme interactions are derived empirically and the various model interactions reproduce the properties of stable nuclei very well the predictions for nuclei far off stability are found to differ drastically. Similar observation are also made for relativistic mean-field theories. Thus, the question arises to what extent the asymmetry, i.e. isospin dependent contributions to the Bethe-Weizsäcker mass formula are really understood.

It is apparent that systematic studies of asymmetric matter are required. In this work we describe asymmetric nuclear matter in the relativistic Brueckner-Hartree-Fock (RBHF) approach. Compared to symmetric matter the theoretical and numerical efforts, respectively, are much larger because protons and neutrons are occupying different Fermi spheres. This implies different effective masses for protons and neutrons and the two sorts of particles cover a different range of off-shell momenta. Hence, it can be expected that beside the isospin also the momentum dependence of self-energies will be of importance. As noted before, asymmetric matter requires to solve a system of coupled equations for the set $T_{\tau_1,\tau_2,\tau_3,\tau_4}$ of in-medium proton-neutron T-matrices, where $\tau = \pm 1$ denote a proton or neutron, respectively. We perform our calculation in a plane-wave spinor basis in which we have neutrons and

protons as distinguishable particles. This implies that we also take into account partial-wave amplitudes that are zero in symmetric nuclear and neutron matter, where the masses are equal and isospin is a good quantum number. It is not clear whether other calculations take these amplitudes into account. Details of the theoretical formulation are discussed in Sect. 2.

A central point of the discussion is the derivation of a new asymmetry dependent invariant. Traditionally, the positive energy-projected in-medium on-shell T-matrix is expressed in terms of five Lorentz invariant amplitudes (see Eq.(2.7)). This standard set of invariants is known to be not unique because each can be replaced by its derivative counterpart. A well known example is the ambiguity found by changing from the pseudo-scalar to the pseudo-vector $\pi - NN$ coupling. A proper treatment avoiding such ambiguities is to solve the in-medium scattering problem for the full Dirac-space including also negative energy states. Such an approach was applied by Huber et al. [8] using the so-called Λ-approximation. In our approach we continue to use the projected T-matrix. To do so we need to introduce a sixth invariant since in asymmetric matter there are six independent helicity amplitudes. Clearly, the new invariant must be defined such that the description of *symmetric* matter is conserved. A promising guideline is to consider an amplitude which approaches zero for equal proton and neutron effective masses. Such an amplitude will vanish in symmetric matter and thus fulfills the constraint mentioned above. In order to account for the differences in $T_{np,np}$ and $T_{np,pn}$ one actually has to define separate invariant amplitudes for the direct and the exchange channel. In the direct channel a vector-scalar and in the exchange channel a pseudovector-pseudoscalar combination is required. However, only the direct amplitude contributes to the proton/neutron self-energies.

Results for the equation of state for nuclear matter at various degrees of asymmetry are discussed in Sect. 3. Particular emphasis is put on the momentum structure of self-energies. We define effective isoscalar and isovector self-energies which directly characterize the mean-field of asymmetric nuclear matter. The dependence of the equation of state on the isovector density is tested and compared to the prediction of the mass formula of being quadratic. The paper closes in Sect. 4 with a summary and conclusions.

II. THEORETICAL FRAMEWORK

The implementation of the RBHF model we use is identical to the one presented in Ref. [4], an account of a calculation for asymmetric nuclear matter can be found in Ref. [6]. We will only briefly review the concepts of the relativistic Brueckner model, but will elaborate further on an omission in the calculation of Ref. [6].

The central quantity in the model is the self-energy, in the RBHF model it has a structure in the spinor representation, expressed by scalar and vector components:

$$\Sigma(p) = \Sigma^s(p) - \gamma^0 \Sigma^0(p) + \bar{\gamma} \cdot \bar{p} \Sigma^v. \tag{2.1}$$

Inserting this form of the self-energy into the Dirac equation, it is natural to define 'effective' quantities by

$$m^* = m_N + \Sigma^s(p),\ p_0^* = p_0 + \Sigma^0(p),\ \bar{p}^* = (1 + \Sigma^v(p))\bar{p}. \tag{2.2}$$

This can be simplified further by dividing out the (small) Σ^v term [3,4]: $m^* = m_N + \Sigma^s(p)/(1+\Sigma^v(p))$. One thus finds a solution of the 'dressed' Dirac equation; the effective spinor:

$$u_r^*(\bar{p}) = \left(\frac{E_p^* + m^*}{2m^*}\right)^{\frac{1}{2}} \begin{pmatrix} 1 \\ \dfrac{\bar{\sigma}\cdot\bar{p}}{E_p^* + m^*} \end{pmatrix} \chi_r, \tag{2.3}$$

with $E^* = \sqrt{\bar{p}^2 + m^{*2}}$. The self-energy is found to depend only weakly on the momentum at least inside the Fermi-sphere: in the practical calculation it is taken constant and set to the value at the Fermi-surface. This greatly simplifies the calculations.

Obviously in asymmetric nuclear matter the Fermi-momenta of protons and neutrons are different, leading to different Pauli-blocking operators and corresponding neutron and the proton self-energies. This asymmetry is also reflected in the fact that one has three different effective interactions, represented by three different T-matrices: $T_{pp,pp}$ (proton-proton scattering), $T_{pn,pn/np}$ (proton-neutron scattering) and $T_{nn,nn}$ (neutron-neutron scattering). In the Brueckner scheme these are calculated by:

$$\begin{aligned}
T_{nn,nn} &= V_{nn,nn} + \int V_{nn,nn} g_{nn} T_{nn,nn} \\
T_{pp,pp} &= V_{pp,pp} + \int V_{pp,pp} g_{pp} T_{pp,pp} \\
T_{np,np} &= V_{np,np} + \int V_{np,np} g_{np} T_{np,np} + \int V_{np,pn} g_{pn} T_{pn,np} \\
T_{pn,np} &= V_{pn,np} + \int V_{pn,np} g_{np} T_{np,np} + \int V_{pn,pn} g_{pn} T_{pn,np}
\end{aligned} \tag{2.4}$$

Note the coupled channels for T_{np}, we also stress that $V_{np,pn}$ is *not* related by a Fierz-transform to $V_{np,pn}$: in $V_{np,pn}$ we have neutron-proton vertices on which only the isospin-1 mesons contribute. It immediately follows that the same holds for the T-matrices: $T_{np,np}$ is not related to $T_{np,pn}$. The 2-body propagators g_{ij} are the usual ones as they appear in the three-dimensionally reduced Thompson equation:

$$g_{ij} = \frac{m_i^*}{E_i^*}\frac{m_j^*}{E_j^*}\frac{\bar{Q}_{ij}(P,\sqrt{s^*},p)}{\sqrt{s^*} - E_i^* - E_j^* + i\epsilon}, \tag{2.5}$$

where $\bar{Q}_{ij}$ is the angle averaged Pauli blocker of the intermediate momentum p, a function of the total momentum P and invariant mass $\sqrt{s^*}$, its parameters depend on the appropriate Fermi-momenta $p_{f,i/j}$. After obtaining a solution for the T-matrices, the self-energy is then calculated using a Brueckner-type expression:

$$\Sigma_N = \int Tr\left[T_{nn}^A g_n\right] + \int Tr\left[T_{np}^A g_p\right], \tag{2.6}$$

where T_{np}^A stands for $T_{np,np}(\theta=0) - T_{np,pn}(\theta=\pi)$. For the proton self-energy the expression is completely similar. We will specify these expressions further later on. For the moment we note that the integrations extend over the neutron and proton Fermi sea, respectively. This calculation is repeated until a self-consistent solution for the self-energy is found. With the

self-consistent self-energy one then can calculate quantities like binding energy and single-particle energies.

In our implementation of the relativistic Brueckner model we calculate the self-energy by decomposing the various T-matrices in Lorentz invariant amplitudes [3]. This boils down to transforming the spin-representation to spinor-representation. The contributions of the various amplitudes to the components of the self-energy are then easily found. In the case of identical particles (to be more precise, particles with equal mass), five amplitudes are needed for the decomposition of the on-shell T-matrix. We use a set of invariants including the pseudo-vector coupling. In the c.m. system we then have:

$$\begin{aligned} \langle p'|T(P,\sqrt{s^*})|p\rangle &= \sum_i \Gamma_i(P,\sqrt{s^*},p',p)\Phi_i \\ \Phi_i &= [\bar{u}^*(\bar{p}')f_i u^*(\bar{p})]\,[\bar{u}^*(-\bar{p}')f_i u^*(-\bar{p})] \\ f_i &\in \{1,\gamma^\mu,\sigma^{\mu\nu},\gamma^5\gamma^\mu,\frac{\not{q}\gamma^5}{2m^*}\}. \end{aligned} \tag{2.7}$$

With q the transferred momentum: $q = p' - p$. For the on-shell scattering matrix elements we have in the c.m. frame $|p'| = |p|$ and $\sqrt{s^*} = E_i^* + E_j^*$. To calculate the self-energy one needs the amplitudes for the direct $(\theta = 0, \bar{p}' = \bar{p})$ and exchange $(\theta = \pi, \bar{p}' = -\bar{p})$ seperately. At these points the set of invariants is degenerate, this is solved by decomposing at small angles and extrapolating to $\theta = 0, \pi$. In Ref. [3] it is shown that this limit exists.

It has often been noted this particular set of invariants is not unique. One can always replace an invariant with its derivative counterpart, we did this by using the pseudo-vector invariant instead of the pseudo-scalar invariant. In this specific case one finds the same coefficient Γ_i since the T-matrix is on-shell, but the contribution to the self-energy is different. However, the contribution to the total energy (found by sandwiching the self-energy between effective spinors) is the same for a given T-matrix. In a self-consistent calculation there will be a difference due to the fact that the self-consistent self-energy will be different, and thus the T-matrix is different. One guideline to limit the freedom in the choice of invariants is to require that one uses the same vertices in the invariant amplitudes as appear in the OBE interaction. Since we have a pseudo-vector coupling for the pion, we thus also have to use the pseudo-vector invariant in the projection.

To avoid this ambiguity one has to calculate the scattering in the complete Dirac space, as is done e.g. in Ref. [8]. For the unique decomposition one needs $\bar{u}^*(\bar{p})\Sigma(\bar{p})u^*(\bar{p})$, $\bar{u}^*(\bar{p})\Sigma(\bar{p})v^*(\bar{p}) = \bar{v}^*(\bar{p})\Sigma(\bar{p})u^*(\bar{p})$ and $\bar{v}^*(\bar{p})\Sigma(\bar{p})v^*(\bar{p})$. In our calculation we only have the amplitude between positive energy spinors (the mean-field generated by the self-energy). Two different decompositions of the self-energy Σ, Σ' need to give the same value for the mean-field: $\bar{u}^*(\bar{p})\Sigma(\bar{p})u^*(\bar{p}) = \bar{u}^*(\bar{p})\Sigma'(\bar{p})u^*(\bar{p})$. One can easily check that up to $\mathcal{O}(p^4/m^{*4})$ this implies for the components $\Sigma'_s = \Sigma_s - \alpha$, $\Sigma'_0 = \Sigma_0 - \alpha$ and $\Sigma'_v = \Sigma_v - \alpha/2m^*$. The 'ambiguity' coefficient α will of course depend on density and momentum. Inserting this in the expression for the renormalized effective mass we see that this is shifted by a quantity much smaller than α. Typically, an error of 100 MeV leads to a shift of ~ 40 MeV in the renormalized effective mass. From the above it follows that any approximate method for the decomposition of the self-energy will still give fair results for quantities that involve only the mean-field, like the binding energy.

In the case of particles having different masses, the helicity matrix elements cease to be symmetric under exchange of particles. Since the number of invariants needed to decompose

the T-matrix is equal to the number of independent helicity matrix elements after exhausting all the symmetries, the loss of this symmetry implies that one needs additional invariants to decompose the T-matrix. Appearantly this was not considered in Ref. [6]. For the on-shell np T-matrix, needed for the calculation of the self-energy, six invariants are required. Although it is relatively easy to find additional invariants that span the six-dimensional space of the six independent helicity matrix elements, the problem is that we need to have a decomposition that reduces to the one used in the symmetric case. With this we mean that the coefficient of the additional, sixth, amplitude should approach zero in the limit of equal masses. Obviously this is not a trivial requirement.

Since $T_{np,np}$ and $T_{np,pn}$ are different we can choose invariants for the exchange and direct amplitudes independently. We encountered a similar situation for the $N\Delta$ scattering matrix [11]. For the present case we found the following invariants which satisfy the above criterium:

$$\begin{aligned}\Phi_6^{dir} &= [\bar{u}_n^*(\bar{p}')\not{k}u_n^*(\bar{p})]\left[\bar{u}_p^*(-\bar{p}')u_p^*(-\bar{p})\right] + [\bar{u}_n^*(\bar{p}')u_n^*(\bar{p})]\left[\bar{u}_p^*(-\bar{p}')\not{k}u_p^*(-\bar{p})\right],\\ \Phi_6^{exch} &= \left[\bar{u}_n^*(\bar{p}')\gamma^5\not{k}u_p^*(\bar{p})\right]\left[\bar{u}_p^*(-\bar{p}')\gamma^5 u_n^*(-\bar{p})\right] + \left[\bar{u}_n^*(\bar{p}')\gamma^5 u_p^*(\bar{p})\right]\left[\bar{u}_p^*(-\bar{p}')\gamma^5\not{k}u_n^*(-\bar{p})\right],\end{aligned} \quad (2.8)$$

where we used standard definitions from scattering theory $k = p_1 - p_4$ which results in $k_0 = E_1 - E_4, \bar{k} = \bar{p} + \bar{p}'$. The numerical indices are the particle numbers (1 and 2 are incoming, 3 and 4 outgoing), the indices n, p denote a neutron and a proton spinor respectively. The amplitude in the exchange channel is antisymmetric under exchange of particles, even when the masses are equal (the other invariants are symmetric under exchange of particles). This does not mean that it is not a valid invariant, but it results in a vanishing coefficient for equal masses, which is exactly what we required. We were unable to find an invariant with this property for the direct channel, but the one given above does have a vanishing coefficient for equal masses. In both invariants the momentum k appears. This makes it impossible to interpret them as some sort of meson channel since a meson carries a momentum $q = p_1 - p_3$. To us there is no straightforward interpretation of these invariants beyond that they are needed to project out the T-matrix in the np channel. This T-matrix spans up a six-dimensional space in spin-space, and we therefore need a six-dimensional basis in spinor space for the basis transformation from spin to spinor representation. We want to stress again that it is crucial for the description of *symmetric* nuclear matter that for asymmetric nuclear matter we have a set of six invariants that behaves properly in the limit of symmetric nuclear matter (equal proton and neutron masses). If we could not find such a set, the method of extracting self-energies in symmetric nuclear matter by means of invariant amplitudes would have an additional ambiguity since the self-energies would not be a continuous function of the asymmetry parameter.

Inserting the decomposition in the expression for the self-energies we find:

$$\begin{aligned}\Sigma_n(p_{f,n}) = &\int_0^{p_{f,n}} \frac{d^3q}{(2\pi)^3}\frac{1}{2E^*_{q,n}}\sum_i \left\{f^i Tr[(\not{q}^* + m_n^*)f^i]\Gamma^i_{dir,nn} - f^i(\not{q}^* + m_n^*)f^i\Gamma^i_{exch,nn}\right\}\\ &+ \int_0^{p_{f,p}} \frac{d^3q}{(2\pi)^3}\frac{1}{2E^*_{q,p}}\sum_i \left\{f^i Tr[(\not{q}^* + m_p^*)f^i]\Gamma^i_{dir,np} - f^i(\not{q}^* + m_p^*)f^i\Gamma^i_{exch,np}\right\}.\end{aligned} \quad (2.9)$$

Working out the Dirac algebra, we find the components of the self-energy expressed in terms of the various $\Gamma^\alpha(s^*, P)$. Taking the Fermi-momentum p_f along the z-axis and transforming

to the c.m.-defining variables $s^* = (p_1 + p_2)^2$ and $P = \bar{p}_1 + \bar{p}_2$ we have for the scalar and vector components of the self-energy at the Fermi-surface

$$\begin{aligned}
&\Sigma^s_n(p_{f,n}) = \\
&\int_0^{4p^2_{f,n}} dP^2 \int_{s^*_{min,n}}^{s^*_{max,n}} ds^* \frac{1}{32\pi^2 p_{f,n}} \frac{m^*_n}{E^*_{p_{f,n}} + E^*_{q,n}} \\
&\times(4\Gamma^s_{dir,nn} - \Gamma^s_{exch,nn} - 4\Gamma^v_{exch,nn} - 12\Gamma^t_{exch,nn} + 4\Gamma^{av}_{exch,nn} + \frac{m^{*\,2}_n - p^\mu_{f,n} q_\mu}{2m^{*\,2}_n} \Gamma^{pv}_{exch,nn}) + \\
&\int_0^{4p^2_{f,p}} dP^2 \int_{s^*_{min,p}}^{s^*_{max,p}} ds^* \frac{1}{32\pi^2 p_{f,n}} \frac{m^*_p}{E^*_{p_{f,n}} + E^*_{q,p}} \\
&\times(4\Gamma^s_{dir,np} + 4\frac{p^\mu_{f,n} q_\mu - m^{*2}_p}{m^*_p} \Gamma^6_{dir,np} - \Gamma^s_{exch,np} - 4\Gamma^v_{exch,np} - 12\Gamma^t_{exch,np} + 4\Gamma^{av}_{exch,np} \\
&\quad + \frac{2(m^{*2}_p - p^\mu_{f,n} q_\mu) - m^{*2}_p + m^{*2}_n}{(m^*_n + m^*_p)^2} \Gamma^{pv}_{exch,np}),
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
&\Sigma^0_n(p_{f,n}) = \\
&\int_0^{4p^2_{f,n}} dP^2 \int_{s^*_{min,n}}^{s^*_{max,n}} ds^* \frac{1}{32\pi^2 p_{f,n}} \frac{E^*_{q,n}}{E^*_{p_{f,n}} + E^*_{q,n}} \\
&\times(-4\Gamma^v_{dir,nn} + \Gamma^s_{exch,nn} - 2\Gamma^v_{exch,nn} - 2\Gamma^{av}_{exch,nn} - \frac{E^*_{p_{f,n}}}{E^*_q} \frac{m^{*\,2}_n - p^\mu_{f,n} q_\mu}{2m^{*\,2}_n} \Gamma^{pv}_{exch,nn}) + \\
&\int_0^{4p^2_{f,p}} dP^2 \int_{s^*_{min,p}}^{s^*_{max,p}} ds^* \frac{1}{32\pi^2 p_{f,n}} \frac{E^*_{q,p}}{E^*_{p_{f,n}} + E^*_{q,p}} \\
&\times(-4\Gamma^v_{dir,np} - 4m^*_p \frac{E^*_{p_{f,n}} - E^*_{q,p}}{E^*_{q,p}} \Gamma^6_{dir,np} + \Gamma^s_{exch,np} - 2\Gamma^v_{exch,np} - 2\Gamma^{av}_{exch,np} \\
&\quad - \frac{2E^*_{p_{f,n}}(m^{*2}_p - p^\mu_{f,n} q_\mu) - E^*_{q,p}(m^{*2}_p - m^{*2}_n)}{E^*_{q,p}(m^*_n + m^*_p)^2} \Gamma^{pv}_{exch,nn})
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
&\Sigma^v_n(p_{f,n}) = \\
&\int_0^{4p^2_{f,n}} dP^2 \int_{s^*_{min,n}}^{s^*_{max,n}} ds^* \frac{1}{32\pi^2 p_{f,n}} \frac{m^*_n}{E^*_{p_{f,n}} + E^*_{q,n}} \\
&\times(-4\Gamma^v_{dir,nn} + \Gamma^s_{exch,nn} - 2\Gamma^v_{exch,nn} - 2\Gamma^{av}_{exch,nn} - \frac{p_{f,n}}{q_z} \frac{m^{*\,2}_n - p^\mu_{f,n} q_\mu}{2m^{*\,2}_n} \Gamma^{pv}_{exch,nn}) + \\
&\int_0^{4p^2_{f,p}} dP^2 \int_{s^*_{min,p}}^{s^*_{max,p}} ds^* \frac{1}{32\pi^2 p_{f,n}} \frac{q_z}{E^*_{p_{f,n}} + E^*_{q,p}} \\
&\times(-4\Gamma^v_{dir,np} - 4m^*_p \frac{p_{f,n} - q_z}{q_z} \Gamma^6_{dir,np} + \Gamma^s_{exch,np} - 2\Gamma^v_{exch,np} - 2\Gamma^{av}_{exch,np} \\
&\quad - \frac{2p_{f,n}(m^{*2}_p - p^\mu_{f,n} q_\mu) - q_z(m^{*2}_p - m^{*2}_n)}{q_z(m^*_n + m^*_p)^2} \Gamma^{pv}_{exch,np}).
\end{aligned} \tag{2.12}$$

q is the momentum of the integrated particle in the nuclear matter rest-frame, found by applying the inverse transform to the set (s^*, P) Also p^μ_f is the on-shell four-momentum of

the incoming particle, taken at the Fermi momentum, so $p^0_{f,i} = E^*_i$. The integration limits are given by $s^*_{min,i} = (E^*_{p_{f,i}} + E^*_{q_{min,i},i})^2 - P^2$ with $q_{min,i} = |P - p_{f,i}|$ and $s^*_{max,i} = 4{E^*_{p_{f,i}}}^2 - P^2$. The index i stands for either a neutron or a proton; $p_{f,i}$ are the respective Fermi-momenta and $E^*_{p,i} = (p^2 + m_i^{*2})^{1/2}$.

The coefficient $\Gamma^6_{exch,np}$ does not appear in the expressions for the self-energy: for $\theta = \pi$, $\bar{k} = 0$ and since the extrapolated $\Gamma^6_{exch,np}$ is finite (we checked this by renormalizing the invariant by dividing by $|k|$) it does not contribute to the self-energy.

III. RESULTS

In Fig. 1 we present the results for the equation of state for various values of the asymmetry parameter Z/A using the Groningen potential [4]. Following Ref. [6] we used an 'averaged' Fermi momentum defined by:

$$\rho = \frac{2}{3\pi^2}\bar{p}_f^3 = \frac{1}{3\pi^2}p^3_{f,p} + \frac{1}{3\pi^2}p^3_{f,n}. \tag{3.1}$$

In the calculation it turns out that the contribution of the sixth invariant is negligible, although it of course indirectly affects the results by influencing the values of the other coefficients. Overall the results look pretty much like the ones presented in Refs. [6–9]. In a way this is not surprising because the results for $Z/A = 0.5$ (nuclear matter) and $Z/A = 0$ (neutron matter) are unaffected by the sixth invariant. The binding energy curves for intermediate values of Z/A will lie between these two extreme curves, which does not provide a large freedom. A more quantitative measure of the equation of state is the asymmetry-energy coefficient a_4, multiplying the $4(Z-A)^2/A$ term in the semi-empirical mass-formula. As has been argued in [6] one should use the binding energies at the various saturation points in the determination of this coefficient and we determine a_4 by

$$a_4 = \frac{1}{4}\frac{\partial^2}{\partial\xi^2}E_b(\xi)\mid_{\xi=Z/A,\rho=\rho_{eq}(Z/A)} . \tag{3.2}$$

Using this prescription we find for the Groningen potential $a_4 = 25$ MeV. Recalculating it for the Bonn C potential [2] we find a sightly larger value of 28 MeV. In the literature one finds a rather wide range of phenomenological values. Older ones tend to give lower values, e.g. in [12] a value of $a_4 = 23.7$ MeV is given, a more recent liquid drop model calculation cites a result in the range 27-30 MeV while a very recent work comes up with $a_4 = 32.65$ MeV [13]. Our result also shows that extracting the asymmetry coefficient via fitting self-energies only for symmetric nuclear and neutron matter, as is done e.g. in Ref. [14], gives much too large results. These authors find values of 35 MeV and higher. We also note that although for the binding energy we find a nice quadratic dependence on the asymmetry parameter, this is not the case for the other quantities like effective masses and self-energies (taken at the respective Fermi-momenta). There we find a dependence which is more linear in character.

In studying the momentum dependence of the self-energies it is convenient to define isoscalar and isovector quantities. For a given observable O we set

$$O_s = \frac{1}{2}(O_n + O_p), O_v = \frac{1}{2}(O_n - O_p), \tag{3.3}$$

In Fig. 2 we show the isoscalar and isovector mean-field as a function of the relative momentum p/p_f, calculated at the respective saturation densities for each asymmetry parameter. As in Refs. [4,5] we define the mean-field as the shift in the pole of the nucleon propagator due to the medium correlations. We see that up to an asymmetry parameter of 0.3 the isovector mean-field is rather independent of Z/A. There are some minor differences, but the slope is essentially the same. Only at $Z/A = 0.2$ we find a deviation, this might be due to the fact that we calculate the mean-field at the saturation density, which is different in this case. The isovector mean-field shows more structure. We observe the intuitively expected increasing difference with increasing asymmetry (corresponding to a lower value of Z/A). However, there is no quadratic dependence on the density A significant dependence on the momentum is found for high momenta, while inside the Fermi-seaa smooth behaviour is obtained.

We show a similar analysis for the isovector components of the scalar and vector self-energy. The isovector scalar and vector self-energies can be interpreted as a 'effective' δ-meson and ρ-meson strengths. Again we see a notable momentum dependence, similar to the one observed in the isovector mean-field.

It is also interesting to analyze our results in terms of mean-field effective coupling constants. This is particulary useful in calculations of finite nuclei [15]. As in the analysis above, we have four channels, both a scalar and vector in either the isoscalar and isovector channel. For this analysis we ignore the momentum dependence and calculate the self-energy of both the neutron and proton at the average Fermi-momentum as defined in Eq. 3.1. The effective coupling constants are then defined by:

$$
\begin{aligned}
\left(\frac{g_\sigma^*}{m_\sigma}\right)^2 &= -\frac{1}{2}\frac{\Sigma_n^s(\bar{p}_f)+\Sigma_p^s(\bar{p}_f)}{\rho_n^s+\rho_p^s} \\
\left(\frac{g_\omega^*}{m_\omega}\right)^2 &= -\frac{1}{2}\frac{\Sigma_n^0(\bar{p}_f)+\Sigma_p^0(\bar{p}_f)}{\rho_n^v+\rho_p^v} \\
\left(\frac{g_\delta^*}{m_\delta}\right)^2 &= -\frac{1}{2}\frac{\Sigma_n^s(\bar{p}_f)-\Sigma_p^s(\bar{p}_f)}{\rho_n^s-\rho_p^s} \\
\left(\frac{g_\rho^*}{m_\rho}\right)^2 &= -\frac{1}{2}\frac{\Sigma_n^0(\bar{p}_f)-\Sigma_p^0(\bar{p}_f)}{\rho_n^v-\rho_p^v},
\end{aligned}
\tag{3.4}
$$

where ρ_i^s and ρ_i^v are the usual scalar and vector densities from relativistic mean-field theory [19]. The results are presented in Fig. 4. As one intuitively expects, the scalar and vector isoscalar channels carry the largest strength. The dependence on the asymmetry parameter is small but we do observe a strong dependence on the density. For the densities presented in the figure, which range from half to double normal nuclear matter density, the dependence is predominantly linear in the Fermi momentum except for the isovector scalar channel. There, a significant dependence on higher orders in p_f and the asymmetry is found. This is in agreement with the finding of Boersma and Malfliet for their density dependent parametrization of the G-matrix. They strongly favour a density dependence of the coupling constants linear in p_f as well [16], over other possibilities like the exponential dependence of Marcos *et al.* [17]. The strength in the isovector channels is smaller although equal in both the scalar and vector channel. The significant scalar strength in the isovector channel suggests that for fits using only a limited number of meson, like the one of Ref. [17] one certainly needs to include

a δ-meson as well. For densities corresponding to $p_f > 0.24$ GeV/c we again find negligible dependence on the asymmetry parameter and a linear dependence on the Fermi momentum. We find a positive sign for g_δ^{*2}, which is not the case when one uses the momentum dependence of the single-particle energy to extract the scalar and vector self-energy components [18]. Note that since our decomposition of the self-energy is not unique the same holds for this analysis in terms of effective coupling constants. However, we think our approximate method still will be able to indicate trends. Still, both the neutron and proton self-energy will have an 'error' $\alpha_{n,p}$. If these are very different due to e.g. a strong dependence on the momentum, one can obtain spurious results in the isovector channels as the negative sign of g_δ^{*2} indicates when using the momentum dependence method.

Below $p_f \sim 0.24$ GeV/c the density dependence changes and an enhancement of the isovector couplings is found at large asymmetry. Most probably this is related to the appearance of bound states related to pairing, which typically appear at low densities in a Brueckner approach. In that sense the Brueckner scheme is an intermediate-density approximation, losing its physical significance at low densities. At low densities the Brueckner independent-pair assumption ceases to be valid because of the onset of the pairing instability and the approach of the deuteron bound-state pole in the *np*-channel. For $Z/A = 0.2$ the proton density is already very low, and one starts to see the onset of pairing correlations. A similar behaviour can be observed in the density dependent parametrization of the former results of Ref. [6] by Marcos *et al.* [17].

The widely used relativistic mean-field theories (RMT) [19,20] are based on Lagrangians including σ, ω and ρ mesons. Density independent meson-nucleon coupling constants obtained phemomenologically from fits to nuclear properties are used. In Tab. I our DB coupling constants, taken at the saturation density ρ_0=0.16 fm^{-3}, are compared to standard density independent values from the σ-ω model (without scalar self-interactions) [19]. For comparison also the density dependent DB isoscalar scalar and vector coupling constants of ref. [21] are shown which in [15] were found to describe nuclear properties very well. In the isoscalar channels the two DB sets are in rather good agreement and both calculations reproduce almost perfectly the purely phenomenological RMT coupling constants. It is also worth noting that in the isoscalar channel the effective coupling constants at saturation density are remarkably close to the 'bare' values as they appear in the interaction. This means, that around these densities the higher order effects generated by the Brueckner ladder are small. We therefore should find only small dependencies on e.g. the asymmetry, which we do observe. In the isovector channel the situation is totally different, there the effective coupling constants are much larger than the bare values. This implies that the (density and asymmetry dependent) higher order effects due to the Brueckner ladders are significant. Indeed, we see a pronounced dependence on asymmetry and density of the effective coupling constants of the effective isovector mesons.

Most important and conclusive for field theoretical models are the results for the isovector channels. The DB and RMT ρ coupling appear to be surprisingly close but is has to be noted that the isovector RMT coupling constants are not well determined. In fact, there exist other RMT parameter sets [19] obtained from fits to finite nuclei with slightly different isoscalar couplings but about twice as large values for g_ρ^2. From Tab. I it is seen that the isovector-scalar and the isovector-vector DB couplings are of comparable strength. This result gives clear evidence that the δ-meson should be included in effective field theories.

The scalar coupling of the δ meson has important consequences for the dynamics in N $\neq$ Z systems. Because protons and neutrons will carry different effective masses they become dynamically distinguishable while the ρ meson leads primarily to energy shifts. Most important for applications to finite nuclei is that isovector effects in the spin-orbit potential will be enhanced as required by single particle spectra in charge asymmetric nuclei. Inclusion of the δ meson might help to resolve the uncertainties in the RMT isovector sector.

IV. CONCLUSIONS

We presented a calculation of asymmetric nuclear matter in a relativistic Brueckner framework. To extract the components of the self-energies we used the method of projecting on Lorentz-invariant amplitudes. We found that for asymmetric nuclear matter a sixth invariant is needed on top of the conventional set of five invariants as for example used in Refs. [4,5]. This sixth invariant has to be chosen in such a way that for an asymmetry parameter close to 0.5 its coefficient has to vanish so that one recovers the usual description of symmetric nuclear matter in terms of five Lorentz-invariant amplitudes. We presented a choice of this sixth invariant that satisfies this criterium. With this model we performed calculations for several observables. The binding energy of the saturation point showed the expected quadratic dependence on the asymmetry parameter. For the asymmetry energy we found a value of around 25 MeV. We analysed our results in terms of mean-field scalar-vector isoscalar-isovector quantities. Apart from the expected scalar-vector strength in the isoscalar channel we also found significant scalar strength in the isovector channel. This can be interpreted as an effective δ-meson, which couples as strong as the effective isovector-vector ρ-meson. Finally, we found a density dependence linear in the Fermi momentum of the effective coupling constants, which was also observed in other calculations [16].

REFERENCES

[1] M.R. Anastasio, L.S. Celenza, W.S. Pong, and C.M. Shakin, Phys. Rep. **100**, 327 (1983).

[2] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

[3] C.J. Horowitz and B.D. Serot, Nucl. Phys. **A464**, 613 (1987).

[4] B. ter Haar and R. Malfliet, Phys. Rep. **149**, 207 (1987); R. Malfliet in *Progress in Particle and Nuclear Physics*, edited by A. Faessler (1988), Vol. 21, p. 207.

[5] Fred de Jong and Rudi Malfliet, Phys. Rev. C **44**, 998 (1991), F. de Jong and H. Lenske, Phys. Rev. C **54**, 1488 (1996).

[6] B. ter Haar and R. Malfliet, Phys. Rev. Lett. **59**, 1652 (1987); R. Malfliet, Nucl. Phys. **A488**, 721c (1988).

[7] L. Engvik, M. Hjorth-Jensen, E. Osnes, G. Bao and E. Oestgaard, Phys. Rev. Lett. **73**, 2650 (1994); L. Engvik, E. Osnes, M. Hjorth-Jensen, G. Bao and E. Oestgaard, Astrophys. Jour. **469**, 794 (1996).

[8] H. Huber, F. Weber, and M.K. Weigel, Phys. Rev. C **51**, 1790 (1995).

[9] C.-H. Lee, T.T.S. Kuo, G.Q. Li, and G.E. Brown, preprint SUNY-NTG-97-10.

[10] see e.g. C.J. Pethick, D. G. Ravenhall, and C.P. Lorentz, Nucl. Phys. **A584**, 675 (1995).

[11] Fred de Jong and Rudi Malfliet, Phys. Rev. C **46**, 2567 (1992), Phys. Rev. C **49**, 1709 (1994).

[12] A.E.S. Green, Phys. Rev. **95**, 1006 (1954); A.E.S. Green and D.F. Edwards, Phys. Rev. **91**, 46 (1953).

[13] W.D. Myers and W.J. Swiatecky, Nucl. Phys. **A601**, 141 (1996).

[14] N.M. Savuskin *et al.*, Phys. Rev. C **55**, 167 (1997).

[15] H. Lenske, C. Fuchs, Phys.Lett.**B 345** 355 (1995); C. Fuchs, H. Lenske, H.H. Wolter, Phys.Rev.**C 52** 3043 (1995).

[16] Hielke Freerk Boersma and Rudi Malfliet, Phys. Rev. C **49** 233 (1994).

[17] S. Marcos, R. Niembro, J.F. Mathiot, N. van Giai, and R. Malfliet, Phys. Rev. C **39** 1134 (1989).

[18] S. Ulrych and H. Muether, preprint, nucl-th/9706030.

[19] B. D. Serot and J.D. Walecka, in *Advances of Nuclear Physics*, edited by J.W. Negele and E. Vogt,(Plenum Press, New York, 1986), **Vol.16**, p. 1.

[20] Y.K. Gambhir, P. Ring, A. Thimet, Ann.Phys. (N.Y.) **198** 132 (1990).

[21] S. Haddad, M.K. Weigel, S. Haddad and M. Weigel, Phys.Rev. **C48** 2740 (1993).

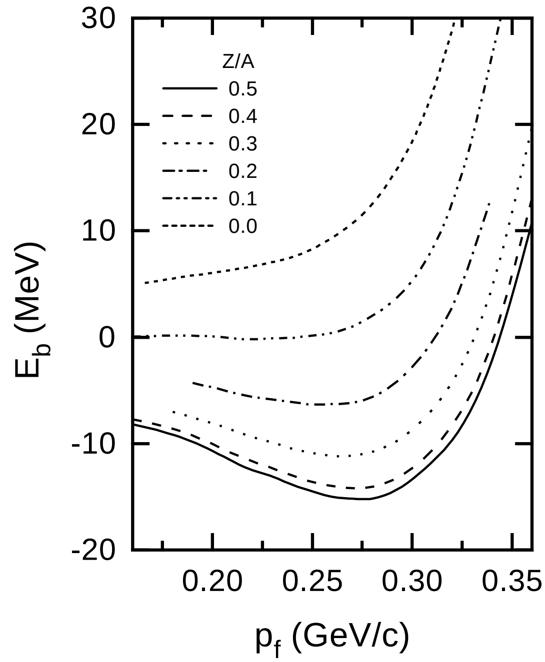


FIG. 1. The binding energy as a function of the density for proton ratios Z/A ranging from 0 (neutron matter) to 0.5 (nuclear matter).

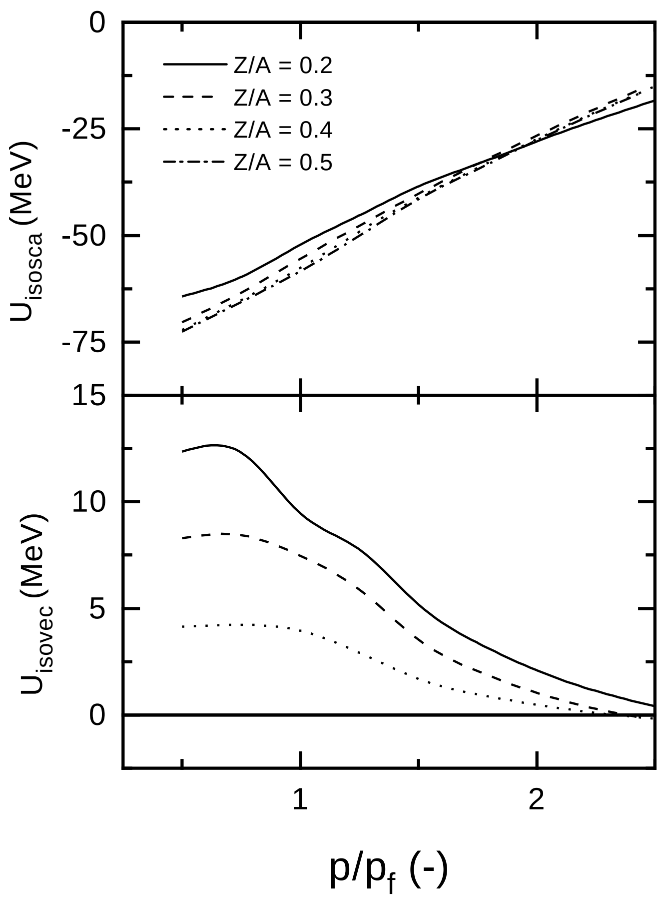


FIG. 2. The iso-scalar and iso-vector components of the mean-field for various asymmetry parameters. The mean-fields are evaluated at the saturation density for the respective asymmetry parameter. The momentum is given relative to the average Fermi-momentum as defined in the text.

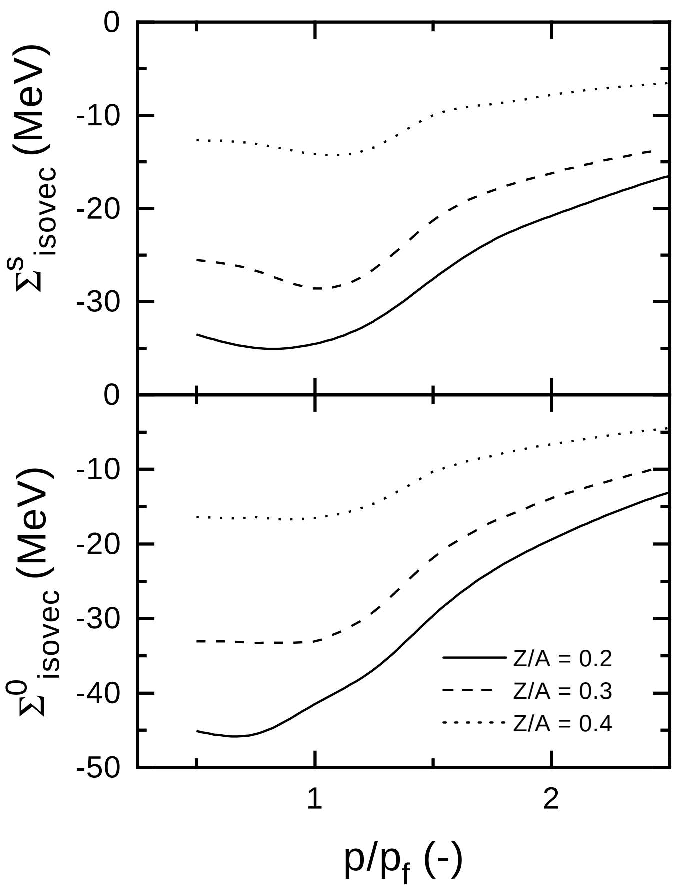
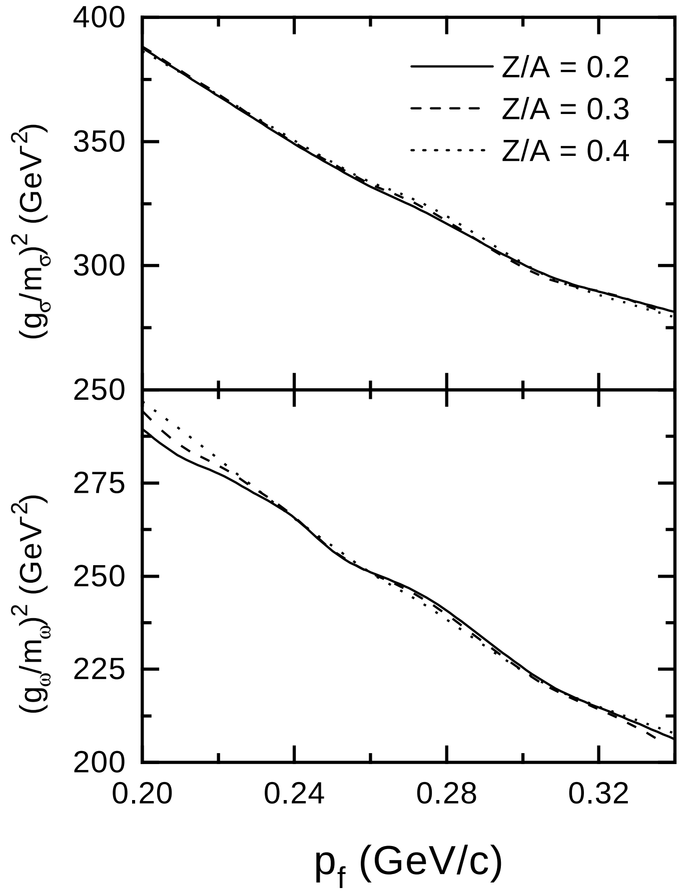


FIG. 3. The isovector components of the scalar and vector components of the self-energy for various asymmetry parameters. These are evaluated at the saturation density for the respective asymmetry parameter. The momentum is given relative to the average Fermi-momentum as defined in the text.


Z/A = 0.2
Z/A = 0.3
Z/A = 0.4
$(g_\sigma/m_\sigma)^2$ (GeV^{-2})
$(g_\omega/m_\omega)^2$ (GeV^{-2})
400
350
300
250
275
250
225
200
0.20
0.24
0.28
0.32
p_f (GeV/c)

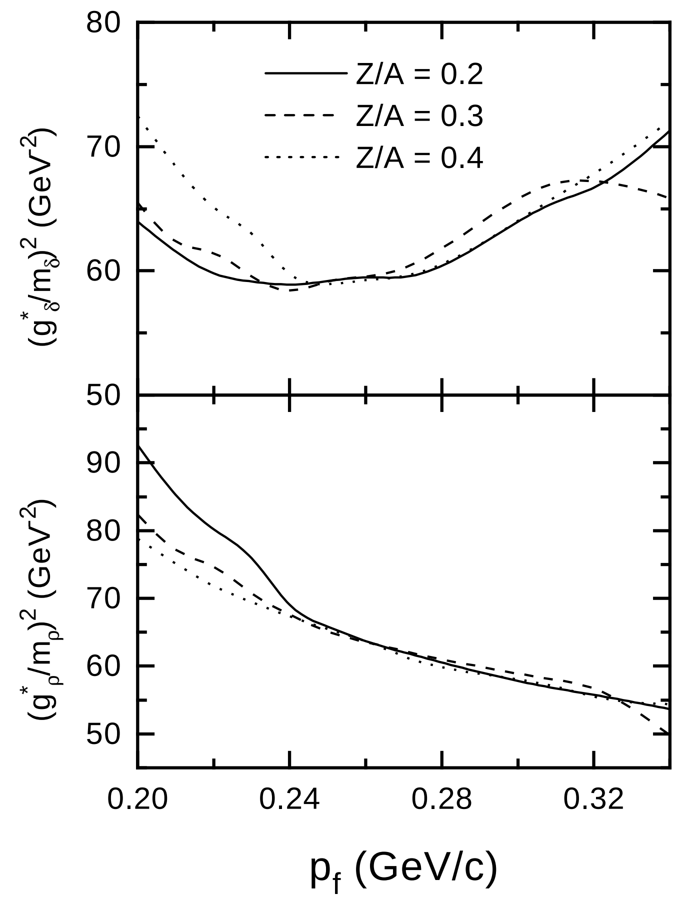


FIG. 4. The effective coupling constants in the isoscalar scalar (g_σ), and vector (g_ω) and the isovector scalar (g_δ) and vector (g_ρ) as defined in the text for various asymmetry parameters and densities.

TABLES

Meson	$g^2_{DB}/4\pi$	$g^2_{HW}/4\pi$	$g^2_{RMT}/4\pi$	Mass [MeV]
σ	7.82	7.51	7.29	550
ω	12.10	11.52	10.84	783
ρ	2.97	-	2.93	770
δ	4.61	-	-	983

TABLE I. DB coupling constants taken at saturation density ρ=0.16 fm^{-3} of the present work (first column) and from ref. [21] (second column, obtained from the Bonn A NN-potential) are compared to RMT values [19] (third column). The meson masses used to derive the coupling constants from the results of Fig.2 are displayed in the last column.